

# Low energy threshold corrections to neutrino masses and mixing angles

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**Abstract.** We compute the low energy threshold corrections to neutrino masses and mixing in the standard model (SM) and its minimal supersymmetric version, using the effective theory technique. We demonstrate that they stabilize the results for neutrino masses and mixing with respect to the choice of the scale to which the renormalization group (RG) equation is integrated. (This confirms the correctness of the recent re-derivation of the RGE for the SM in hep-ph/0108005.) Since, as is known, those corrections are potentially very important for phenomenology we derive for them the explicit formulae that can be applied to specific models of neutrino masses and mixing.

## 1 Introduction

There is at present strong experimental evidence for neutrino oscillations. Their most natural explanation is the existence of neutrino masses. Neutrino masses can be incorporated in the standard model (SM) or its supersymmetric extension (MSSM) by adding to the Lagrangian the non-renormalizable dimension-5 operator [1]

$$\Delta\mathcal{L}_{\text{SM}} = -\frac{1}{4M}\mathbf{C}^{AB}(\epsilon_{ki}H_k l_i^A)(\epsilon_{lj}H_l l_j^B) + \text{H.c.}, \quad (1.1)$$

in which  $A, B = 1, 2, 3$  label generations,  $l_j^A = (\nu^A, e^A)$  are the Weyl spinors transforming as doublets of  $SU(2)_L$ ,  $H_i$  is the Higgs doublet with hypercharge  $+1/2$  and  $\epsilon_{21} = -\epsilon_{12} = 1$ ,  $\epsilon_{ii} = 0$ . After electroweak symmetry breaking (1.1) gives the neutrino mass matrix in the form

$$(m_\nu^{\text{tree}})^{AB} = \frac{1}{4M}\mathbf{C}^{AB}v^2, \quad (1.2)$$

where  $v$  is the vacuum expectation value of the neutral component of the Higgs field  $H_i$ ; this is diagonalized by the unitary rotation  $\nu_A \rightarrow U^{Aa}\nu_a$ . The elements of the matrix  $U$  determine the neutrino oscillation probabilities and are, therefore, probed in the neutrino experiments.

The operator (1.1) appears in the low energy effective theory as a result of integrating out fields of an underlying theory describing physics at some high energy scale. Thus, it is assumed to be generated at some high scale  $M_F$  (much higher than the electroweak scale  $M_Z$ ). Therefore obtaining reliable predictions for neutrino masses and mixing angles requires solving the renormalization group equation (RGE) for the Wilson coefficient  $\mathbf{C}^{AB}$  [2,3]

$$\frac{d}{dt}\mathbf{C}^{AB} = K\mathbf{C}^{AB} + \kappa\left[y_{e_A}^2\mathbf{C}^{AB} + \mathbf{C}^{AB}y_{e_B}^2\right], \quad (1.3)$$

where  $t = (1/16\pi^2)\ln(Q/M_Z)$  and  $y_{e_A}^2$  are the Yukawa couplings of the charged leptons<sup>1</sup>. In the SM  $\kappa = -3/2$  and  $K = -3g_2^2 + 2\sum_{\text{fermions}}N_c^{(f)}y_f^2 + 2\lambda$  where  $N_c^{(f)} = 3$  for quarks and 1 for leptons; in the MSSM  $\kappa = +1$  and  $K = -6g_2^2 - 2g_Y^2 + 6\sum_A y_{\nu_A}^2$  with  $g_Y^2 = (3/5)g_1^2$ .

The solution to (1.3) [4],

$$\mathbf{C}(Q) = I_K \mathcal{J} \mathbf{C}(M_F) \mathcal{J}, \quad (1.4)$$

where  $\mathcal{J} = \text{diag}(I_e, I_\mu, I_\tau)$  and

$$I_K = \exp\left(-\int_0^{t_Q} K(t')dt'\right),$$

$$I_{e_A} = \exp\left(-\kappa\int_0^{t_Q} y_{e_A}^2(t')dt'\right) \approx 1 - I_A^{\text{rg}}, \quad (1.5)$$

with  $t_Q = (1/16\pi^2)\ln(M_F/Q)$ , gives  $\mathbf{C}(Q)$  at the electroweak scale  $Q \approx M_Z$  in terms of  $\mathbf{C}(M_F)$ . The RGE (1.3) was analyzed in many papers [5] to see how much the initial pattern of neutrino masses and mixing angles generated at the scale  $M_F$  is modified by quantum corrections involving large logarithms  $\ln(M_F/M_Z) \gg 1$ . In particular, it has been found [6] that (1.3) exhibits a non-trivial fixed point structure. In many interesting cases (e.g. for degenerate or partially degenerate neutrino mass spectrum) that structure leads to a pattern of mixing angles that is not compatible with the present experimental indications

<sup>1</sup> Without loss of generality throughout this paper we work in the basis in which the matrix of the charged lepton Yukawa couplings is diagonal (this basis is preserved by the RG evolution of the charged lepton Yukawa couplings below the scale  $M_F$ ); the Wilson coefficient  $\mathbf{C}^{AB}$  is therefore assumed to be given in that basis too. The normalization of  $\lambda$  is fixed by the Higgs self-interaction:  $\mathcal{L}_{\text{self}} = -(\lambda/2)(H^\dagger H)^2$

(i.e. bimaximal mixing and small  $U_{13}$  matrix element). It has however been pointed out [8–10] that in the MSSM the so-called low energy threshold corrections which were neglected in previous analyses [5,6] can in some cases be more important than the RG evolution and can change qualitatively the pattern obtained by solving (1.3).

In this paper we compute these low energy threshold corrections both in the SM and in the MSSM. We first show that they stabilize the results for neutrino masses and mixing with respect to the choice of the low energy scale  $Q$  to which the RGE is integrated out (to clarify the points raised in the recently published paper [11] we demonstrate this explicitly in a pedagogical way) and assess their magnitude and dependence on the parameters of the MSSM.

## 2 Standard model

In this section we calculate one-loop corrections to the neutrino mass matrix in the SM. Our starting point is the SM Lagrangian (see e.g. [12]) supplemented with the non-renormalizable term (1.1). All parameters of this Lagrangian are understood to be running parameters renormalized at the scale  $Q \sim M_Z$ . Also the Wilson coefficient  $C^{AB}$  of the dimension-5  $\Delta L = 2$  operator (1.1) is a renormalized parameter of the effective theory Lagrangian. Integrating its RGE (1.3) from the high scale  $M_F$  down to some scale  $Q \approx M_Z$  resums potentially large corrections involving  $\ln(M_F/Q)$  to all orders of the perturbation expansion. However, since the low energy scale  $Q$  is not a priori determined by any physical requirement (apart from the condition  $Q \sim M_Z$ ), the neutrino masses and mixing angles computed in the tree-level approximation from the Wilson coefficient  $C^{AB}(Q)$  do depend (albeit weakly) on the actual choice of  $Q$ . This dependence can be removed by computing masses and mixing angles in the one-loop approximation in the  $\overline{\text{MS}}$  scheme with the same renormalization scale  $Q$ .

Since the neutrino masses are orders of magnitude smaller than the electroweak scale, the calculation of the low energy threshold corrections is technically most easily achieved in the effective theory approach. At the scale  $Q \approx M_Z$  all gauge and Higgs bosons are integrated out and the effective theory valid below the electroweak scale is constructed. In this low energy theory the one-loop neutrino mass matrix  $(m_\nu^{1\text{-loop}})^{AB}$  is given by the tree-level term (1.2) of the SM plus the one-loop (threshold) correction  $\Delta m_\nu$ . The latter, apart from having a leading  $\ln Q$  dependence that exactly matches the  $\ln Q$  dependence of the tree-level mass  $(m_\nu^{\text{tree}})^{AB}(Q)$  (1.2), can also contain non-trivial  $Q$ -independent pieces.

Writing the SM Higgs doublet as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}G^+ \\ v + \phi^0 + iG^0 \end{pmatrix}, \quad (2.1)$$

we get from (1.1) the neutrino mass term and various interactions (we write down only those which will be rele-



**Fig. 1.** One-particle irreducible threshold corrections

vant for us):

$$\begin{aligned} \Delta \mathcal{L}_{\text{SM}} = & -\frac{1}{2} (m_\nu^{\text{tree}})^{AB} \nu_A \nu_B \\ & + \frac{v C^{AB}}{2\sqrt{2}M} G^+ e_B \nu_A - \frac{v C^{AB}}{4M} \phi^0 \nu_B \nu_A \\ & - \frac{C^{AB}}{8M} (\phi^0 \phi^0 - G^0 G^0) \nu_B \nu_A + \text{H.c.} \quad (2.2) \end{aligned}$$

In principle, to get the Feynman rules for the neutrino mass eigenstates  $\nu_a$  one has also to rotate the neutrino fields  $\nu_A \rightarrow U^{Aa} \nu_a$  (where  $U^{Aa}$  is the matrix diagonalizing  $(m_\nu^{\text{tree}})^{AB}$ ). This step is however unnecessary for our purpose since we will everywhere use massless neutrino propagators on internal lines<sup>2</sup>. We can therefore compute directly the corrections to the tree-level mass matrix  $C^{AB}$  in the basis in which it is not necessarily diagonal.

The strategy is now to integrate out heavy fields:  $W^\pm$ ,  $Z^0$ ,  $\phi^0$  (as well as Goldstone bosons  $G^\pm$  and  $G^0$ ) and to construct the effective Lagrangian valid below the scale  $Q \sim M_Z$ . Up to terms of order  $\mathcal{O}(p^2/M_Z^2)$  where  $p \sim m_\nu \ll M_Z$  is the external four momentum, one-loop effects (shown schematically in Fig. 1) of the heavy fields present in the full SM have to be simulated by the corrections  $\delta z^{AB}$  and  $\delta m_\nu^{AB}$  in the effective theory Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \bar{\nu}_A (\delta^{AB} + \delta z^{AB}) i \bar{\sigma}^\mu \partial_\mu \nu_B \\ & - \frac{1}{2} \left[ (m_\nu^{\text{tree}} + \delta m_\nu)^{AB} \nu_A \nu_B + \text{H.c.} \right] + \dots \quad (2.3) \end{aligned}$$

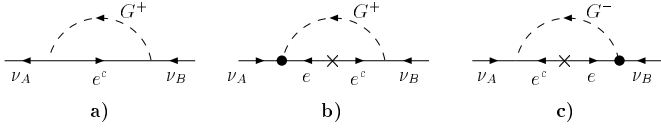
Redefining the neutrino fields to get their kinetic term canonical and using  $\delta z^{AB} = -\Sigma_V^{AB}(0)$ ,  $\delta m_\nu^{AB} = \Sigma_m^{AB}(0)$  (where  $\Sigma_V^{AB}(p^2)$  and  $\Sigma_m^{AB}(p^2)$  are defined in Fig. 1) one gets

$$\begin{aligned} (\Delta m_\nu)^{AB} = & I_{A'A}^{\text{th}} (m_\nu^{\text{tree}})^{A'B} + (m_\nu^{\text{tree}})^{AB'} I_{B'B}^{\text{th}}, \\ I_{AB}^{\text{th}} \equiv & \frac{1}{2} \Sigma_V^{AB}(0) + \frac{1}{2} \Delta^{AB}, \quad (2.4) \end{aligned}$$

where we have split

$$\Sigma_m^{AB}(0) = \frac{1}{2} \Delta^{A'A} (m_\nu^{\text{tree}})^{A'B} + \frac{1}{2} (m_\nu^{\text{tree}})^{AB'} \Delta^{B'B}. \quad (2.5)$$

<sup>2</sup> Taking non-zero neutrino masses into account in propagators would amount to including  $1/M^2$  effects (where  $M$  is the mass scale of the heavy neutrino states). Since we do not consider operators of dimension higher than five resulting from the seesaw mechanism we cannot compute  $1/M^2$  effects consistently



**Fig. 2a–c.** Contributions of the charged Goldstone boson. Heavy dots indicate vertices arising from the operator (1.1). Crosses indicate fermion propagators with a helicity flip (i.e. with the fermion mass in the numerator)

The  $G^\pm$  Goldstone bosons' contribution to  $\Sigma_V^{AB}$  and  $\Sigma_m^{AB}$  are shown in Figs. 2a,b,c, respectively, where the dots denote the interaction vertex originating from  $\Delta\mathcal{L}_{\text{SM}}$  given in (2.2). In a general  $R_\xi$ -gauge and working in the  $\overline{\text{MS}}$  scheme one finds

$$\begin{aligned}\Sigma_V^{AB}(0) &= \frac{1}{2}\delta^{AB}y_{e_A}^2 \\ &\quad \times [(m_{e_A}^2 - \xi_W M_W^2)B'_0(e_A, G^\pm) + B_0(e_A, G^\pm)], \\ \Sigma_m^{AB}(0) &= -\frac{v^2}{4M} \\ &\quad \times [y_{e_A}^2 \mathbf{C}^{AB} B_0(e_A, G^\pm) + \mathbf{C}^{AB} y_{e_B}^2 B_0(e_B, G^\pm)].\end{aligned}\quad (2.6)$$

We have used the abbreviated notation for the standard two-point function

$$\begin{aligned}B_0(1, 2) &\equiv B_0(0, m_1, m_2) \\ &= \frac{1}{(4\pi)^2} \left[ -1 + \frac{m_1^2}{m_1^2 - m_2^2} \ln \frac{m_1^2}{Q^2} + \frac{m_2^2}{m_2^2 - m_1^2} \ln \frac{m_2^2}{Q^2} \right], \\ B'_0(1, 2) &\equiv \left. \frac{d}{dp^2} B_0(p^2, m_1, m_2) \right|_{p^2=0} \\ &= \frac{1}{(4\pi)^2} \left[ -\frac{1}{2} \frac{m_1^2 + m_2^2}{(m_1^2 - m_2^2)^2} + \frac{m_1^2 m_2^2}{(m_1^2 - m_2^2)^3} \ln \frac{m_1^2}{m_2^2} \right],\end{aligned}$$

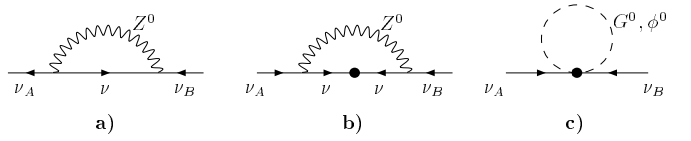
where  $m_1$  and  $m_2$  are the masses of particles 1 and 2.

The  $W^\pm$  boson exchange contributes only to  $\Sigma_V$ :

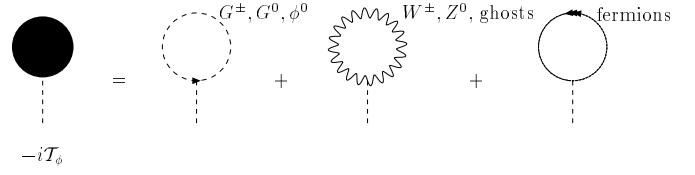
$$\begin{aligned}\Sigma_V^{AB} &= \frac{g_2^2}{2}\delta^{AB} \left[ (m_{e_A}^2 - M_W^2)B'_0(e_A, W^\pm) \right. \\ &\quad \left. + B_0(e_A, W^\pm) + 1 \right] + \frac{g_2^2}{4}\delta^{AB} \\ &\quad \times \left\{ \frac{m_{e_A}^2}{M_W^2} \left[ (m_{e_A}^2 - M_W^2)B'_0(e_A, W^\pm) \right. \right. \\ &\quad \left. \left. - B_0(e_A, W^\pm) \right] - 2B_0(e_A, W^\pm) \right. \\ &\quad \left. - \frac{m_{e_A}^2}{M_W^2} \left[ (m_{e_A}^2 - \xi_W M_W^2)B'_0(e_A, G^\pm) \right. \right. \\ &\quad \left. \left. - B_0(e_A, G^\pm) \right] + 2\xi_W B_0(e_A, G^\pm) \right\}.\end{aligned}\quad (2.7)$$

Together, the  $G^\pm$  and  $W^\pm$  contributions give

$$I_{AB}^{W^\pm G^\pm} = \delta^{AB} \frac{g_2^2}{2} \frac{m_{e_B}^2}{M_W^2} \frac{1}{4} \left[ (m_{e_B}^2 - M_W^2)B'_0(e_B, W^\pm) \right.$$



**Fig. 3a–c.** Contributions of  $Z^0$  and neutral scalars



**Fig. 4.** Tadpole diagrams

$$\begin{aligned}& -3B_0(e_B, W^\pm) \Big] + \delta^{AB} \frac{g_2^2}{4} \left[ (m_{e_B}^2 - M_W^2) \right. \\ & \quad \left. \times B'_0(e_B, W^\pm) + B_0(e_B, W^\pm) + 1 \right] \\ & - \delta^{AB} \frac{g_2^2}{4} \left[ \left( 1 - \frac{m_{e_B}^2}{M_W^2} \right) B_0(e_B, W^\pm) \right. \\ & \quad \left. \times \left( \xi_W - \frac{m_{e_B}^2}{M_W^2} \right) B_0(e_B, G^\pm) \right].\end{aligned}\quad (2.8)$$

In the limit of massless neutrinos on internal lines, the contribution of  $Z^0$  exchange to  $\Sigma_V$  arising from the diagram shown in Fig. 3a can be obtained from (2.8) by setting there  $m_{e_B} = 0$  and replacing  $W^\pm(G^\pm) \rightarrow Z^0(G^0)$ ,  $g_2^2 \rightarrow (g_2^2 + g_Y^2)/2$ . This gives

$$\begin{aligned}I_{AB}^{Z^0(1)} &= \delta^{AB} \frac{g_2^2 + g_Y^2}{8} \\ &\quad \times [1 - M_Z^2 B'_0(\nu, Z^0) + \xi_Z B_0(\nu, G^0)].\end{aligned}\quad (2.9)$$

(We do not write any index on  $\nu$  to stress that neutrino masses are set to zero in the  $B_0$  functions.) The diagram shown in Fig. 3b, which arises due to the non-zero Majorana mass insertion, contributes to  $\Sigma_m^{AB}(0)$ . It gives

$$\begin{aligned}I_{AB}^{Z^0(2)} &= \delta^{AB} \frac{g_2^2 + g_Y^2}{2} \\ &\quad \times \left[ B_0(\nu, Z^0) + \frac{1}{4M_Z^2} a(G^0) - \frac{1}{4M_Z^2} a(Z^0) \right].\end{aligned}\quad (2.10)$$

Finally the exchange of  $G^0$  and  $\phi^0$  in the diagram shown in Fig. 3c gives

$$I_{G^0, \phi^0}^{AB} = \delta^{AB} \frac{1}{2v^2} [a(\phi^0) - a(G^0)],\quad (2.11)$$

where  $16\pi^2 a = m^2[-1 + \ln(m^2/Q^2)]$  is another standard loop function. The  $\xi_Z$  dependent part of this contribution cancels the  $\xi_Z$  dependence of (2.10).

The combined contribution of (2.8)–(2.11) is still gauge dependent because one has to include the contribution of tadpole diagrams shown in Fig. 4. They give

$$I_{AB}^{\mathcal{T}} = -\delta^{AB} \frac{\mathcal{T}_\phi}{M_\phi^2 v} = -\delta^{AB} \frac{\mathcal{T}_\phi}{\lambda v^3},$$

$$\begin{aligned}
&= \frac{\delta^{AB}}{16\pi^2} \left\{ \frac{g^2}{4} \xi_W \left( 1 - \ln \frac{\xi_W M_W^2}{Q^2} \right) \right. \\
&+ \frac{g^2 + g_Y^2}{8} \xi_Z \left( 1 - \ln \frac{\xi_Z M_W^2}{Q^2} \right) \\
&- \frac{3}{2} \lambda \left( -1 + \ln \frac{M_\phi^2}{Q^2} \right) \\
&+ \frac{1}{\lambda} \sum_{f,A} N_c^{(f)} y_{fA}^4 \left( -1 + \ln \frac{m_{fA}^2}{Q^2} \right) \\
&- \frac{1}{\lambda} \left[ \frac{3}{8} g^4 \left( -\frac{1}{3} + \ln \frac{M_W^2}{Q^2} \right) \right. \\
&\left. + \frac{3}{16} (g_2^2 + g_Y^2)^2 \left( -\frac{1}{3} + \ln \frac{M_Z^2}{Q^2} \right) \right] \Big\}. \quad (2.12)
\end{aligned}$$

It is easy to see that the  $\xi_W$  and  $\xi_Z$  dependence of (2.8) and (2.9) is canceled by (2.12).

To check that  $(m_\nu^{1-\text{loop}})^{AB}$  is independent of the renormalization scale  $Q$  we must recall the RGE for the vacuum expectation value  $v^2$ . Since in this approach  $v^2$  is merely an abbreviation for  $-2m^2/\lambda$  where  $m^2$  and  $\lambda$  are the (negative) mass squared parameter and the self-coupling of the Higgs doublet, respectively, we have

$$\frac{d}{dt} v^2 = v^2 \left( \frac{1}{m^2} \frac{d}{dt} m^2 - \frac{1}{\lambda} \frac{d}{dt} \lambda \right), \quad (2.13)$$

where  $16\pi^2 t = \ln Q$ . The RG equations for  $m^2$  and  $\lambda$  read

$$\begin{aligned}
\frac{d}{dt} m^2 &= m^2 \left( -\frac{9}{2} g_2^2 - \frac{3}{2} g_Y^2 + 6\lambda + 2T \right), \\
\frac{d}{dt} \lambda &= 12\lambda^2 - (9g_2^2 + g_Y^2)\lambda + \frac{9}{4} g^4 + \frac{3}{2} g_2^2 g_Y^2 + \frac{3}{4} g_Y^4 \\
&\quad + 4\lambda T - 4T_2,
\end{aligned}$$

where  $T \equiv \sum_f N_c^{(f)} y_f^2$  and  $T_2 \equiv \sum_f N_c^{(f)} y_f^4$ . Combining (2.13) with the RGE (1.3) for  $\mathbf{C}^{AB}$  we have therefore

$$\begin{aligned}
\frac{1}{4M} \mathbf{C}^{AB}(Q) v^2(Q) &= \frac{1}{4M} \mathbf{C}^{AB}(Q') v^2(Q') \\
&- \frac{3}{2} \frac{v^2}{4M} \left( \mathbf{C}^{AB} y_{eB}^2 + y_{eA}^2 \mathbf{C}^{AB} \right) \ln \frac{Q}{Q'} \\
&- \frac{v^2}{4M} \mathbf{C}^{AB} 4\lambda \ln \frac{Q}{Q'} + \frac{v^2}{4M} \mathbf{C}^{AB} \frac{3}{2} (g_2^2 + g_Y^2) \ln \frac{Q}{Q'} \\
&- \frac{1}{\lambda} \left( \frac{9}{4} g^4 + \frac{3}{2} g_2^2 g_Y^2 + \frac{3}{4} g_Y^4 - 4T_2 \right) \ln \frac{Q}{Q'} + \dots \quad (2.14)
\end{aligned}$$

It is now easy to see that the leading  $\ln Q$  dependence of the tree-level neutrino mass matrix  $(m_\nu^{\text{tree}})^{AB}(Q)$  cancels the explicit  $\ln Q$  dependence of the one-loop correction (2.4). In particular, the  $1/\lambda$  terms in (2.14) cancel the  $\ln Q$  dependence of the  $W^\pm$ ,  $Z^0$  (and their ghosts) and fermionic tadpoles in (2.12).

It is also possible to consider  $v$  not as the tree-level VEV of the Higgs field but instead as the minimum of the full one-loop effective potential. In such an approach

there are no tadpoles<sup>3</sup> but since the effective potential gives finite  $v$  only in the Landau gauge,  $\xi_W = \xi_Z = 0$ , the contributions (2.8)–(2.11) must be taken in this gauge too. In this approach the RGE for  $v_{1-\text{loop}}^2$  is no longer given by (2.13) but instead is determined from the anomalous dimension (also taken in the Landau gauge) of the Higgs field operator:

$$\frac{d}{dt} v_{1-\text{loop}}^2 = v_{1-\text{loop}}^2 \left( \frac{9}{4} g_2^2 + \frac{3}{2} g_Y^2 - 2T \right). \quad (2.15)$$

It is then easy to check that again the explicit  $\ln Q$  dependence of  $(m_\nu^{1-\text{loop}})^{AB}$  obtained from (2.8)–(2.11) in the Landau gauge cancels against the  $\ln Q$  dependence of the tree-level mass matrix  $(1/4M) \mathbf{C}^{AB}(Q) v_{1-\text{loop}}^2(Q)$ .

In practice, the difference between the two approaches (which formally is a higher order effect) is not seen when  $v^2$  (or  $v_{1-\text{loop}}^2$ ) is expressed in terms of the physical  $Z^0$  boson mass. For example, in the first approach one has

$$v^2 = \frac{4(M_Z^2)_{\text{ph}}}{g_2^2 + g_Y^2} \left[ 1 - \frac{\hat{\Pi}_{ZZ}(M_Z^2, Q)}{(M_Z^2)_{\text{ph}}} + \frac{2\mathcal{T}_\phi}{\lambda v^3} \right]. \quad (2.16)$$

where  $\hat{\Pi}_{ZZ}(M_Z^2, Q)$  is the 1-PI self energy of the  $Z^0$  boson computed for  $q^2 = M_Z^2$  and renormalized in the  $\overline{\text{MS}}$  scheme with the renormalization scale  $Q$ .

Neglecting terms of order  $\mathcal{O}(m_{eA}^4/M_W^4)$  and higher the final formula reads (tadpoles have canceled out)

$$\begin{aligned}
I^{AB} &= \frac{\delta^{AB}}{16\pi^2} \left\{ y_{eB}^2 \left( \frac{11}{8} - \frac{3}{4} \ln \frac{M_W^2}{Q^2} \right) \right. \\
&+ \frac{g^2}{4} \left( \frac{1}{2} + \ln \frac{M_W^2}{Q^2} \right) + \frac{g^2 + g_Y^2}{8} \left( -\frac{5}{2} + 4 \ln \frac{M_Z^2}{Q^2} \right) \\
&\left. + \frac{\lambda}{2} \left( -1 + \ln \frac{M_\phi^2}{Q^2} \right) - 8\pi^2 \frac{\hat{\Pi}_{ZZ}(M_Z^2, Q)}{(M_Z^2)_{\text{ph}}} \right\}, \quad (2.17)
\end{aligned}$$

where we have adopted  $\xi_{W,Z} = 1$  and  $v^2$  in the tree-level neutrino mass matrix is now given by

$$v^2 \equiv \frac{\hat{s}_W^2 \hat{c}_W^2}{\pi \hat{\alpha}_{\text{EM}}} (M_Z^2)_{\text{ph}},$$

where  $\hat{s}^2$  and  $\hat{\alpha}_{\text{EM}}$  are the sinus of the Weinberg angle and fine structure constant, respectively, in the  $\overline{\text{MS}}$  scheme and at the renormalization scale  $Q$  (for which one can take  $M_Z^4$ ). Note also that the factor  $-(3/2) \ln(M_W/Q)$  in the first line of (2.17) confirms the correctness of the recent re-derivation [3] of the SM RGE.

<sup>3</sup> Using  $v$  determined from the full one-loop potential is equivalent to saying that one expands the symmetric Lagrangian around some initially unspecified  $v$  and determines the value of  $v$  from the requirement that the tree-level tadpole (arising from a term in the Lagrangian that is linear in the Higgs field) cancels the one-loop one

<sup>4</sup> In the standard way (see e.g. [12])  $\hat{s}^2(M_Z)$  and  $\hat{\alpha}_{\text{EM}}(M_Z)$  can be expressed in terms of measurable quantities:  $\alpha_{\text{EM}}$  measured in the Thomson scattering and by the Fermi constant  $G_F$

From our discussion it should be clear that putting a particular emphasis on better stability with the renormalization scale  $Q$  of the product  $v^2 \mathbf{C}^{AB}$  (or of its eigenvalues) as in ref. [11] makes no sense in the quantum field theory. The physical neutrino masses defined as the poles of the propagators [15] (or, in the one-loop approximation, as the appropriate coefficients of the effective Lagrangian (2.3)) do not depend on the renormalization scale  $Q$ . On the other hand, by themselves big changes of  $\mathbf{C}^{AB}$  during the RG evolution between  $M_F$  and  $M_Z$  do not signal any instability. They reflect only the importance of the resummation of large logarithmic contributions  $(y_\tau^2 \ln(M_F/M_Z))^n$  where  $n = 1, 2, \dots$  in order to get reliable results for neutrino masses in terms of the Lagrangian parameters defined at the scale  $M_F$ . Finally, let us notice that  $v^2(Q)$  disappears from the final formula (2.17) for neutrino masses. Therefore, the question whether  $v^2$  is considered as an abbreviation for  $-m^2/\lambda$  (in which case its variation with  $Q$  is very rapid) or as the minimum of the full one-loop effective potential is inessential for the stabilization of the results for physical neutrino masses.

### 3 The MSSM

In this section we calculate one-loop corrections to neutrino mass matrix in the MSSM. We will see that, apart from stabilizing the results obtained from the RG analysis with respect to small changes of the final scale  $Q$ , they contain also  $\ln Q$ -independent terms which can be more important than the RG evolution. In some situations [8, 10, 9] they can change the pattern of mixing and lead to relations between the mixing angles different from the one obtained at the infrared fixed point of the RGE (1.3) [6]. We will use the notation and conventions of [14] in which the Feynman rules resulting from the renormalizable part of the MSSM (i.e. without the higher dimension operators) are collected.

The neutrino masses arise in supersymmetric models from one of the dimension-5 operators obtained by adding to the Lagrangian supersymmetric non-renormalizable terms

$$\begin{aligned} \Delta \mathcal{L} &\propto \frac{1}{M} \mathbf{C}^{AB} \int d^2\theta \left( \epsilon_{ij} \hat{H}_i^{(u)} \hat{L}_j^A \right) \left( \epsilon_{lk} \hat{H}_l^{(u)} \hat{L}_k^B \right) + \text{H.c.} \\ &= -\frac{1}{4M} \mathbf{C}^{AB} \left[ \left( \epsilon_{ij} H_i^{(u)} l_j^A \right) \left( \epsilon_{ij} H_i^{(u)} l_j^B \right) \right. \\ &\quad + \left( \epsilon_{ij} h_i^{(u)} L_j^A \right) \left( \epsilon_{ij} h_i^{(u)} L_j^B \right) + 2 \left( \epsilon_{ij} H_i^{(u)} L_j^A \right) \\ &\quad \times \left. \left( \epsilon_{ij} h_i^{(u)} l_j^B \right) + 2 \left( \epsilon_{ij} h_i^{(u)} L_j^A \right) \left( \epsilon_{ij} H_i^{(u)} l_j^B \right) \right] + \text{H.c.} \\ &\quad + \text{terms with auxiliary fields,} \end{aligned} \quad (3.1)$$

where capital letters with a hat denote superfields and capital letters and lower case letters denote their scalar and fermionic components, respectively. In the second line we have fixed the normalization so that the first term coincides with the operator (1.1) in the SM.

Expressing the initial fields in terms of the physical ones as in [14] one gets the following terms (we write down only those which are relevant for our calculation):

$$\begin{aligned} \Delta \mathcal{L}_{\text{MSSM}} &= -\frac{1}{2} (m_\nu^{\text{tree}})^{AB} \nu_A \nu_B, \\ &\quad + \frac{v_u \mathbf{C}^{AB}}{2\sqrt{2}M} Z_H^{2k} H_k^+ e_B \nu_A - \frac{v_u \mathbf{C}^{AB}}{4M} Z_R^{2k} H_k^0 \nu_B \nu_A \\ &\quad - \frac{\mathbf{C}^{AB}}{8M} Z_R^{2i} Z_R^{2j} H_i^0 H_j^0 \nu_B \nu_A \\ &\quad + \frac{\mathbf{C}^{AB}}{8M} Z_H^{2i} Z_H^{2j} H_{i+2}^0 H_{j+2}^0 \nu_B \nu_A \\ &\quad - \frac{v_u \mathbf{C}^{AB}}{\sqrt{2}M} Z^{AJ} Z_N^{4i} \tilde{\nu}_J \chi_i^0 \nu_B \\ &\quad + \frac{v_u \mathbf{C}^{AB}}{2\sqrt{2}M} Z_L^{Ak*} Z_+^{2j} L_k^- \chi_j^+ \nu_B + \text{H.c.} \end{aligned} \quad (3.2)$$

As previously we use Weyl spinors here. From (3.2) the necessary additional Feynman rules can be easily obtained.

The contribution to the quantity  $I^{AB}$  defined in (2.4) of the  $W^\pm$  and  $G^\pm$  bosons is the same as in the SM and is given by (2.8)<sup>5</sup>. The contribution of  $H^\pm$  (arising from diagrams similar to the ones shown in Fig. 2) is

$$\begin{aligned} I_{AB}^{H^\pm} &= \delta^{AB} \frac{g_2^2 m_{e_B}^2}{8 M_W^2} \tan^2 \beta \\ &\quad \times \left[ (m_{e_B}^2 - M_{H^\pm}^2) B_0'(e_B, H^\pm) + B_0(e_B, H^\pm) \right] \\ &\quad + \delta^{AB} \frac{g_2^2 m_{e_B}^2}{2 M_W^2} B_0(e_B, H^\pm), \end{aligned} \quad (3.3)$$

where  $\tan \beta \equiv v_u/v_d$  is the usual ratio of the VEVs of the two Higgs doublet  $H^{(u)}$  and  $H^{(d)}$ .

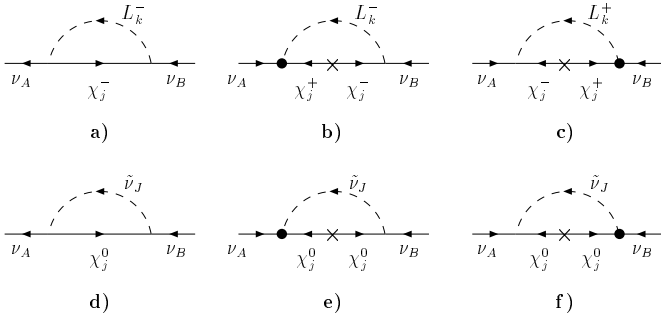
The contribution of  $Z^0$  is the same as in the SM and is given by (2.9, 2.10) while the contribution of neutral scalars (2.11) is in the MSSM replaced by

$$\begin{aligned} I_{AB}^{\text{scalars}} &= \delta^{AB} \frac{1}{2v_u^2} \left[ \sin^2 \alpha a(H^0) + \cos^2 \alpha a(h^0) \right. \\ &\quad \left. - \cos^2 \beta a(A^0) - \sin^2 \beta a(G^0) \right]. \end{aligned} \quad (3.4)$$

Since  $v_u^2/\sin^2 \beta = v_u^2 + v_d^2$  the last term of (3.4) cancels the  $\xi_Z$  dependence in (2.10) as in the SM. The dependence of (2.8) and (2.9) on  $\xi_W$  and  $\xi_Z$ , respectively is again canceled out by the tadpole diagrams with  $G^\pm$  and  $G^0$  loops<sup>6</sup>.

<sup>5</sup> Strictly speaking, in supersymmetry one has to use the DRED scheme [13] instead of DIMREG used in the previous section. This would amount to omitting factors of 1 in the brackets in the second line of (2.8) and in (2.9) and to similar changes in the tadpole contributions. These changes do not affect, however, the interesting part of the threshold corrections which is not proportional to  $\delta^{AB}$

<sup>6</sup> In the MSSM (2.12) is replaced by  $-\delta^{AB} [\mathcal{T}_{h^0} \cos \alpha / M_h^2 + \mathcal{T}_{H^0} \sin \alpha / M_H^2] / v_u$ . This can be brought to a more convenient form by using the tree-level relations:  $\cos^2 \alpha / M_h^2 + \sin^2 \alpha / M_H^2 = (\sin^2 \beta / M_Z^2 + \cos^2 \beta / M_A^2) / \cos^2 2\beta$  and  $\sin \alpha \cos \alpha (1/M_h^2 - 1/M_H^2) = -(\sin \beta \cos \beta / \cos^2 2\beta) (1/M_Z^2 + 1/M_A^2)$



**Fig. 5a–f.** Contributions of charginos/charged sleptons **a–c** and neutralinos/sneutrinos **d–f**

Feynman diagrams describing contributions of chargino/charged slepton and neutralino/sneutrino sectors are shown in Fig. 5. They give

$$\begin{aligned}
I_{AB}^{\text{charg}} &= \frac{1}{4} \left( g_2 Z_L^{Ak*} Z_-^{1j*} + y_{e_A} Z_L^{3+Ak*} Z_-^{2j*} \right) \\
&\times \left( g_2 Z_L^{Bk} Z_-^{1j} + y_{e_B} Z_L^{3+Bk} Z_-^{2j} \right), \\
&\times \left[ (m_{C_j}^2 - M_{L_k}^2) B'_0(C_j, E_k^\pm) + B_0(C_j, E_k^\pm) \right] \\
&- \frac{\sqrt{2}}{v_u} Z_L^{Ak*} Z_+^{2j} \left( g_2 Z_L^{Bk} Z_-^{1j} + y_{e_B} Z_L^{3+Bk} Z_-^{2j} \right) \\
&\times m_{C_j} B_0(C_j, E_k^\pm), \quad (3.5)
\end{aligned}$$

and

$$\begin{aligned}
I_{AB}^{\text{neutr}} &= \frac{g_2^2 + g_Y^2}{8} Z_\nu^{AJ} Z_\nu^{BJ*} \left| s_W Z_N^{1j} - c_W Z_N^{2j} \right|^2 \\
&\times \left[ (m_{N_j}^2 - M_{\tilde{\nu}_j}^2) B'_0(N_j, \tilde{\nu}_j) + B_0(N_j, \tilde{\nu}_j) \right] \\
&- \frac{2}{v_u} \sqrt{g_2^2 + g_Y^2} Z_\nu^{AJ} Z_\nu^{BJ*} Z_N^{4j} \left( s_W Z_N^{1j} - c_W Z_N^{2j} \right) \\
&\times m_{N_j} B_0(N_j, \tilde{\nu}_j). \quad (3.6)
\end{aligned}$$

To check that the  $\ln Q$  dependence of the correction to the neutrino mass matrix in the MSSM matches the one following from the RGE it is more convenient to use the second approach described in the previous section and to assume that  $v_u^2$  in the tree-level neutrino mass matrix is determined from the full one-loop effective potential. Using then its RGE [16]

$$\frac{d}{dt} (v_u^2)_{1\text{-loop}} = (v_u^2)_{1\text{-loop}} \left[ \frac{3}{2} g_2^2 + \frac{1}{2} g_Y^2 - 6 \sum_A y_{u_A}^2 \right] \quad (3.7)$$

and the MSSM RGE for  $C^{AB}$  (1.3) one finds

$$\begin{aligned}
\left[ \frac{v_{1\text{-loop}}^2}{4M} C^{AB} \right] (Q) &= \left[ \frac{v_{1\text{-loop}}^2}{4M} C^{AB} \right] (Q') \\
&- \frac{v_{1\text{-loop}}^2}{4M} C^{AB} \left( \frac{9}{2} g_2^2 + \frac{3}{2} g_Y^2 \right) \ln \frac{Q}{Q'} \\
&+ \frac{v_{1\text{-loop}}^2}{4M} \left( C^{AB} y_{e_B}^2 + y_{e_A}^2 C^{AB} \right) \ln \frac{Q}{Q'}. \quad (3.8)
\end{aligned}$$

It is then easy to see that in the gauge  $\xi_W = \xi_Z = 0$  (in which the effective potential and hence also  $v_{1\text{-loop}}$  is defined) the dependence on  $\ln Q$  in the sum of corrections (2.8)–(2.10) and (3.3)–(3.6) cancels the  $\ln Q$  dependence in (3.8).

The final formula for the factor  $I_{AB}^{\text{th}}$  in the MSSM takes the form

$$\begin{aligned}
16\pi^2 I_{AB}^{\text{th}} &= \delta^{AB} \frac{g^2 m_{e_B}^2}{2 M_W^2} \left\{ \frac{1}{4} (1 + \tan^2 \beta) \right. \\
&\times \left( -\frac{1}{2} + \ln \frac{M_{H^\pm}^2}{Q^2} \right) + \frac{1}{2} \left( 1 + \frac{3}{2} \ln \frac{M_{H^\pm}^2}{M_W^2} \right) \left. \right\} \\
&+ \frac{1}{4} \left( g_2 Z_L^{Ak*} Z_-^{1j*} + y_{e_A} Z_L^{3+Ak*} Z_-^{2j*} \right) \\
&\times \left( g_2 Z_L^{Bk} Z_-^{1j} + y_{e_B} Z_L^{3+Bk} Z_-^{2j} \right) \\
&\times \left[ \ln \frac{M_{E_k^\pm}^2}{Q^2} + f(m_{C_j}^2, M_{E_k^\pm}^2) \right] \\
&- \frac{\sqrt{2}}{v_u} Z_L^{Ak*} Z_+^{2j} \left( g_2 Z_L^{Bk} Z_-^{1j} + y_{e_B} Z_L^{3+Bk} Z_-^{2j} \right) m_{C_j} \\
&\times \left[ \ln \frac{M_{E_k^\pm}^2}{Q^2} + g(m_{C_j}^2, M_{E_k^\pm}^2) \right] \\
&+ \frac{g_2^2 + g_Y^2}{8} Z_\nu^{AJ} Z_\nu^{BJ*} \left| s_W Z_N^{1j} - c_W Z_N^{2j} \right|^2 \\
&\times \left[ \ln \frac{M_{\tilde{\nu}_j}^2}{Q^2} + f(m_{N_j}^2, M_{\tilde{\nu}_j}^2) \right] \\
&- \frac{2}{v_u} \sqrt{g_2^2 + g_Y^2} Z_\nu^{AJ} Z_\nu^{BJ*} Z_N^{4j} \left( s_W Z_N^{1j} - c_W Z_N^{2j} \right) m_{N_j} \\
&\times \left[ \ln \frac{M_{\tilde{\nu}_j}^2}{Q^2} + g(m_{N_j}^2, M_{\tilde{\nu}_j}^2) \right] \\
&+ \text{terms proportional to } \delta^{AB}, \quad (3.9)
\end{aligned}$$

where the functions  $f$  and  $g$  are

$$\begin{aligned}
f(a, b) &= -\frac{1}{2} + \frac{a}{b-a} + \frac{a^2}{(b-a)^2} \ln \frac{a}{b}, \\
g(a, b) &= -1 - \frac{a}{b-a} \ln \frac{a}{b},
\end{aligned}$$

and satisfy  $f(a, a) = g(a, a) = 0$ . The terms proportional to the unit matrix are not interesting as they change only the overall scale of the neutrino masses and do not influence the mixing angles.

Consider now the simplest limit  $M_{H^\pm} = M_{L_k^\pm} = M_{\tilde{\nu}_j} = m_{C_j} = m_{N_j} \equiv M_S$ . Using the solution (1.4) and writing for  $Q \approx M_Z$  the factors  $I_{e_A}^{\text{rg}}$  as

$$\begin{aligned}
I_{e_A} &= \exp \left( - \int_0^{t_Q} y_{e_A}^2(t') dt' \right) \\
&\approx \exp \left( - \int_0^{t_S} y_{e_A}^2(t') dt' \right) \left[ 1 - \frac{y_{e_A}^2}{16\pi^2} \ln \frac{M_S}{Q} \right], \quad (3.10)
\end{aligned}$$

where  $t_Q = (1/16\pi^2) \ln(M_F/Q)$ ,  $t_S = (1/16\pi^2) \ln(M_F/M_S)$  and  $y_{e_A}^2 = (g_2^2/2)(m_{e_A}^2/M_W^2)(1 + \tan^2 \beta)$ , we have (up to the overall normalization)

$$\begin{aligned}
& (m_\nu^{\text{tree}})^{AB}(Q) \\
& \propto (m_\nu^{\text{tree}})^{AB}(M_S) - \frac{1}{16\pi^2} \ln \frac{M_S}{Q} \\
& \quad \times \left[ y_{e_A}^2 (m_\nu^{\text{tree}})^{AB}(M_S) + (m_\nu^{\text{tree}})^{AB}(M_S) y_{e_B}^2 \right] \\
& \quad + \mathcal{O} \left( y_{e_A}^4 \ln^2 \frac{M_S}{Q} \right). \tag{3.11}
\end{aligned}$$

Adding the one-loop correction in the form

$$\begin{aligned}
16\pi^2 I^{AB} &= \delta^{AB} \frac{g^2 m_{e_B}^2}{2 M_W^2} \left\{ \frac{1}{4} (1 + \tan^2 \beta) \left( -\frac{1}{2} + \ln \frac{M_S^2}{Q^2} \right) \right. \\
& \quad \left. + \frac{1}{2} \left( 1 + \frac{3}{2} \ln \frac{M_S^2}{M_W^2} \right) \right\} + \frac{1}{4} \delta^{AB} y_{e_B}^2 \ln \frac{M_S}{Q} \\
& \quad + \text{terms proportional to } \delta^{AB}, \tag{3.12}
\end{aligned}$$

we recover, as far as the logarithms are concerned, the same result that is obtained with the running using the MSSM RGE from the scale  $M_F$  down to  $M_{\text{SUSY}}$ , and then the SM RGE to run down to the  $M_W$  scale [7]. There is however a non-trivial extra non-logarithmic piece

$$\delta^{AB} \frac{g^2 m_{e_B}^2}{2 M_W^2} \left( \frac{3}{8} - \frac{1}{8} \tan^2 \beta \right).$$

which is missed by the RG procedure just described. This piece is usually less important than the effects of the evolution from the scale  $M_F$  down to  $M_{\text{SUSY}}$  but for large  $\tan \beta$  it is more important than the running from  $M_{\text{SUSY}}$  to the  $M_W$  scale with the SM RGE.

## 4 Numerical analysis

In the SM the most important correction which changes the matrix structure of the neutrino mass matrix can be incorporated by substituting

$$\begin{aligned}
\mathbf{C}^{AB}(Q) &\rightarrow \mathbf{C}_{\text{corrected}}^{AB} \\
&\equiv \mathbf{C}^{AB}(Q) + I_A^{\text{th}} \mathbf{C}^{AB}(Q) + \mathbf{C}^{AB}(Q) I_B^{\text{th}}, \tag{4.1}
\end{aligned}$$

where

$$\begin{aligned}
I_A^{\text{th}} &= \frac{1}{16\pi^2} \frac{g_2^2 m_{e_A}^2}{2 M_W^2} \\
& \quad \times \left[ \frac{11}{8} - \frac{3}{2} \ln \frac{M_W}{Q} + \mathcal{O} \left( \frac{m_{e_A}^2}{M_W^2} \ln \frac{m_{e_A}^2}{M_W^2} \right) \right], \tag{4.2}
\end{aligned}$$

in the tree-level formulae for neutrino masses and mixing angles. Since  $I_A^{\text{th}}$  are proportional to the Yukawa couplings  $y_{e_A}^2$ , this correction cannot change qualitatively the results obtained by integrating the RGE and can be most easily taken into account by stopping the RG evolution of the Wilson coefficient  $\mathbf{C}^{AB}(Q)$  at the scale  $Q = M_W e^{-11/12}$ . The remaining corrections affect only the overall scale of the neutrino masses and therefore are not interesting in view of the unspecified magnitude of the mass  $M$  in (1.2).

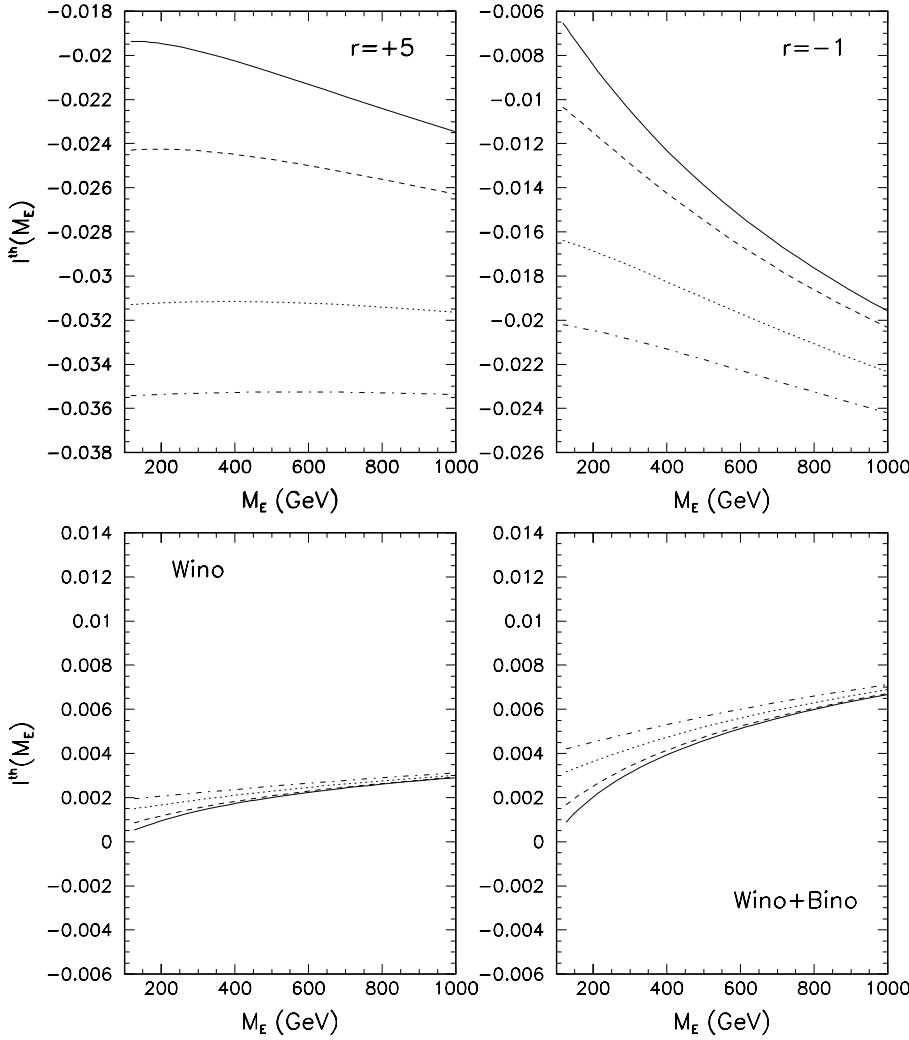
In the MSSM the contribution of  $W^\pm$  and  $H^\pm$  to  $I_{AB}^{\text{th}}$  is also proportional to  $\delta^{AB} m_{e_A}^2 / M_W^2$  and cannot change qualitatively the results of the RG evolution<sup>7</sup>. However the effects of the genuinely supersymmetric contribution  $I_{AB}^{\text{susy}} = I_{AB}^{\text{charg}} + I_{AB}^{\text{neutr}}$  to  $I_{AB}^{\text{th}}$  can be important because unlike the SM case, it is not necessarily proportional to  $\delta^{AB} m_{e_A}^2 / M_W^2$ . It has been demonstrated [8–10], that  $I_{AB}^{\text{susy}}$ , if large, can lead to a relation between the mixing angles different than the one obtained at the infrared fixed point of the RGE [6]. The numerical estimates made in [8–10] relied however on approximating the corrections  $I_{AB}^{\text{th}}$  by the pure wino contribution to  $\Sigma_V^{AB}(0)$ . Here we analyze the dependence of  $I_{AB}^{\text{th}}$  on the MSSM parameters using the full expression (3.9). For simplicity we will assume that the mixing of the left- and right-handed charged sleptons is negligible.

We begin by considering flavor conserving slepton mass matrices. In this case  $I_{AB}^{\text{susy}} = \delta^{AB} I_A^{\text{susy}}$  because the matrices  $Z_L^{Ak}$  and  $Z_\nu^{AJ}$  are diagonal in the generation space. With no mixing of the left- and right-handed charged sleptons, the chargino and neutralino contribution can be simplified to

$$\begin{aligned}
16\pi^2 I_A^{\text{susy}} &= \frac{1}{4} g_2^2 \left| Z_-^{1j} \right|^2 \left[ \ln \frac{M_{E_{LA}^\pm}^2}{Q^2} + f(m_{C_j}^2, M_{E_{LA}^\pm}^2) \right] \\
& \quad + \frac{g_2^2 + g_Y^2}{8} \left| s_W Z_N^{1j} - c_W Z_N^{2j} \right|^2 \\
& \quad \times \left[ \ln \frac{M_{\tilde{\nu}_A}^2}{Q^2} + f(m_{N_j}^2, M_{\tilde{\nu}_A}^2) \right] \\
& \quad - \frac{\sqrt{2}}{v_u} g_2 Z_+^{2j} Z_-^{1j} m_{C_j} \left[ \ln \frac{M_{E_{LA}^\pm}^2}{Q^2} + g(m_{C_j}^2, M_{E_{LA}^\pm}^2) \right] \\
& \quad - \frac{2}{v_u} \sqrt{g_2^2 + g_Y^2} Z_N^{4j} \left( s_W Z_N^{1j} - c_W Z_N^{2j} \right) \\
& \quad \times m_{N_j} \left[ \ln \frac{M_{\tilde{\nu}_A}^2}{Q^2} + g(m_{N_j}^2, M_{\tilde{\nu}_A}^2) \right] \\
& \quad + \frac{1}{4} y_{e_A}^2 \left| Z_-^{2j} \right|^2 \left[ \ln \frac{M_{E_{RA}^\pm}^2}{Q^2} + f(m_{C_j}^2, M_{E_{RA}^\pm}^2) \right], \tag{4.3}
\end{aligned}$$

where  $M_{E_{LA}^\pm}$  and  $M_{E_{RA}^\pm}$  are the masses of the  $A$ th generation left- and right-handed charged sleptons, respectively. The contribution of the right-handed charged sleptons  $(I_A^{\text{susy}})_R$  (last line of (4.3)) is again proportional to the corresponding Yukawa coupling  $y_{e_A}^2$ . Hence, it too only slightly changes the effects of the RG evolution and can be neglected here. The remaining part  $(I_A^{\text{susy}})_L$  of (4.3) depends on the mass of the charged slepton (the mass of the sneutrino is related to it by the underlying  $SU(2)_L$  symmetry:  $M_{\tilde{\nu}_A}^2 = M_{E_{LA}^\pm}^2 + \cos 2\beta M_W^2$ ),  $\tan \beta$  and the parameters of the chargino/neutralino sector:  $\mu$  and  $M_2$  (as is customary, in the neutralino sector we take  $M_1 \approx 0.5 M_2$ ). Figure 6 shows the results of the numerical evaluation of  $(I_A^{\text{susy}})_L \equiv I_L^{\text{susy}}(M_{E_{LA}^\pm})$  and compares it

<sup>7</sup> The same result can be always obtained from the running by slightly changing the value of  $\tan \beta$



**Fig. 6.** Correction  $I_L^{\text{susy}}(M_E)$  as a function of the left-handed charged slepton mass for chargino mass 150 (solid), 250 (dashed), 500 (dotted) and 800 (dot-dashed lines) for  $\tan\beta = 2$  and  $r \equiv M_2/\mu = +5$  and  $-1$ . In lower panels we show the results of retaining only the wino (left) and wino and bino (right) contributions

to the results of approximating  $I^{\text{susy}}(M_{E_{L,A}^\pm})$  by the pure  $\tilde{W}^\pm$  (charged wino) or  $\tilde{W}^\pm, \tilde{W}^0$  and  $\tilde{B}$  (bino) contribution. The striking difference between the complete and approximate calculation is mainly due to the contribution of diagrams 5b,c,e,f which give a negative contribution (third and fourth lines in (4.3)) but are missed in the approximation. Although the absolute magnitude of the correction  $I^{\text{susy}}(M_{E_{L,A}^\pm})$  does depend on  $\tan\beta$  and the chargino composition, the differences  $I^{\text{susy}}(M_{E_{L,A}^\pm}) - I^{\text{susy}}(M_{E_{L,B}^\pm})$  (which are relevant for the changes of the neutrino mass matrix structure) are much less dependent on  $\tan\beta$ . They are however sensitive to the chargino (and neutralino) composition as is clear from the comparison of the two upper panels of Fig. 6.

In [9] it has been observed, that if<sup>8</sup>  $|I_e^{\text{susy}}| \gg |I_\mu^{\text{susy}}|$  and  $I_\mu^{\text{susy}} - (I_\tau^{\text{susy}} - I_\tau^{\text{rg}}) \neq 0$ , the masses of three neutrinos

<sup>8</sup> The renormalization group corrections (1.4) and (1.5) can easily be incorporated in the formula (4.1) by substituting  $I_A^{\text{th}} \rightarrow I_A^{\text{th}} - I_A^{\text{rg}}$ . One then has

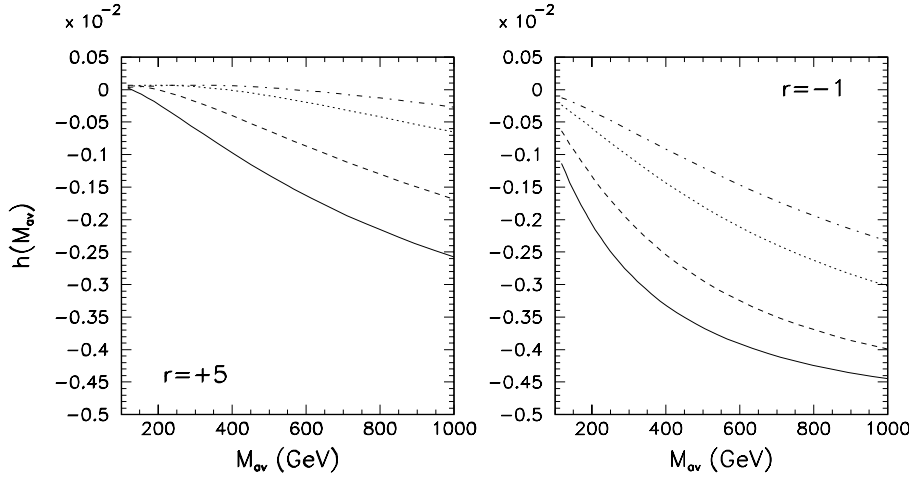
$$\begin{aligned} \mathcal{C}_{\text{corrected}}^{AB} \approx & \mathcal{C}^{AB}(M_F) + [I_A^{\text{th}}(Q) - I_A^{\text{rg}}(M_F/Q)] \mathcal{C}^{AB}(M_F) \\ & + \mathcal{C}^{AB}(M_F) [I_B^{\text{th}}(Q) - I_B^{\text{rg}}(M_F/Q)] \end{aligned}$$

equal at the scale  $M_F$  can be split in agreement with the experimental information, provided the solar mixing angle is (very close to) maximal. For this mechanism to work  $|I_e^{\text{susy}} - I_\mu^{\text{susy}}| \sim 10^{-3}$  is required. We can now improve the estimates made in [9] on the basis of the wino approximation. From Fig. 6 it is clear that for  $M_2/\mu \approx -1$  and lighter chargino mass  $\sim 150$  GeV the mass splitting  $M_{\tilde{\mu}_L} \approx 1.2M_{\tilde{e}_L}$  ( $M_{\tilde{e}_L} \approx 1.2M_{\tilde{\mu}_L}$ ) is sufficient to obtain  $I_e^{\text{susy}} - I_\mu^{\text{susy}} \sim 10^{-3}$  ( $\sim -10^{-3}$ ). For heavier charginos and/or  $M_2/\mu$  positive obtaining  $|I_e^{\text{susy}} - I_\mu^{\text{susy}}| \sim 10^{-3}$  requires very large mass splitting (or is even impossible to achieve).

From the formula (4.3) we can also quantify the magnitude of the off-diagonal corrections  $I_{AB}^{\text{th}}$  induced by the flavor mixing in the slepton mass matrices. Assuming that left-handed charged slepton masses are all approximately equal (which implies that sneutrino masses are also all approximately equal), this is most easily done in the so-called mass insertion approximation [17]. In (4.3) terms involving e.g. charged left-handed sleptons can be written as

$$Z_L^{Ak} Z_L^{Bk*} H(M_{E_k}^2)$$





**Fig. 7.** Coefficient of the mass insertion  $\delta_{LL}^{AB}$  as a function of the left-handed charged slepton mass for chargino mass 150 (solid), 250 (dashed), 500 (dotted) and 800 (dot-dashed lines) for  $\tan\beta = 2$  and  $r \equiv M_2/\mu = -1$  and  $+5$

$$\begin{aligned}
&= Z_L^{Ak} Z_L^{Bk*} H(M_{\text{av}}^2 + (M_{E_k^\pm}^2 - M_{E_{\text{av}}^\pm}^2)) \\
&\approx \delta^{AB} \left[ H(M_{E_{\text{av}}^\pm}^2) - M_{E_{\text{av}}^\pm}^2 H'(M_{E_{\text{av}}^\pm}^2) \right] \\
&+ Z_L^{Ak} M_{E_k^\pm}^2 Z_L^{Bk*} H'(M_{E_{\text{av}}^\pm}^2) \\
&\approx (M_{LL}^2)_{AB} H'(M_{E_{\text{av}}^\pm}^2) + \text{terms proportional to } \delta^{AB},
\end{aligned} \tag{4.4}$$

where  $M_{\text{av}}$  is the common mass of the left-handed charged sleptons and  $H$  is some function. We have also used the defining property of the matrices  $Z_L$ . The mass insertions  $\delta_{LL}^A B$  are then defined as

$$\delta_{LL}^{AB} = \frac{M_{LL}^2}{M_{\text{av}}^2} \quad \text{for } A \neq B. \tag{4.5}$$

In the case of flavor mixing only in the left-handed slepton sector<sup>9</sup> we get for  $I_{AB}^{\text{susy}}$

$$\begin{aligned}
I_{AB}^{\text{susy}} &= \frac{\delta_{LL}^{AB}}{16\pi^2} \left\{ \frac{1}{4} g_2^2 |Z_-^{1j}|^2 F(m_{C_j}^2, M_{E_{\text{av}}^\pm}^2) \right. \\
&+ \frac{g_2^2 + g_Y^2}{8} |s_W Z_N^{1j} - c_W Z_N^{2j}|^2 F(m_{N_j}^2, M_{\tilde{\nu}_{\text{av}}}^2) \\
&- \frac{\sqrt{2}}{v_u} g_2 Z_+^{2j} Z_-^{1j} m_{C_j} G(m_{C_j}^2, M_{E_{\text{av}}^\pm}^2) \\
&- \frac{2}{v_u} \sqrt{g_2^2 + g_Y^2} Z_N^{4j} (s_W Z_N^{1j} - c_W Z_N^{2j}) \\
&\left. \times m_{N_j} G(m_{N_j}^2, M_{\tilde{\nu}_{\text{av}}}^2) \right\},
\end{aligned} \tag{4.6}$$

where

$$\begin{aligned}
F(a, b) &= \frac{b^2 - 3ab}{(b-a)^2} - \frac{2a^2b}{(b-a)^3} \ln \frac{a}{b}, \\
G(a, b) &= \frac{b}{b-a} + \frac{ab}{(b-a)^2} \ln \frac{a}{b}.
\end{aligned} \tag{4.7}$$

<sup>9</sup> Flavor mixing in the right-handed slepton sector gives  $I_{AB}^{\text{susy}}$  suppressed by  $y_{e_A} y_{e_B}$ . Flavor non-diagonal entries in the soft supersymmetry breaking terms mixing charged left- and right-handed sleptons give  $I_{AB}^{\text{susy}}$  proportional to  $g_2 y_{e_B}$  and, hence, substantial only for large  $\tan\beta$  values, i.e. only when the renormalization group corrections are dominant

We have used the fact that because of the underlying  $SU(2)_L$  symmetry, mass insertions in the left-handed charged slepton sector and in the sneutrino sector are the same (and  $M_{E_{\text{av}}^\pm}^2$  and  $M_{\tilde{\nu}_{\text{av}}}^2$  are related). In Fig. 7 we plot the coefficient of  $\delta_{LL}^{AB}$  as a function of  $M_{E_{\text{av}}^\pm}^2$  for several values of the chargino masses for  $\tan\beta = 2$ . We see that for a fixed  $\delta_{LL}^{AB}$ , the biggest values of  $I_{AB}^{\text{susy}}$  are obtained for  $M_2/\mu \approx -1$  and for rather large slepton to chargino mass ratio. In principle the mass insertion approximation should fail for  $|\delta_{XY}^{AB}| \lesssim 0.1$ . In practice it works as an order of magnitude estimate even for  $|\delta_{XY}^{AB}| \lesssim 1$  (the error is then of order 25%). More accurate results can be always obtained from the general formula (3.9).

## 5 Conclusions

We have computed the low energy threshold corrections to the neutrino masses and mixings in the SM and in the MSSM. We have explicitly demonstrated that they are gauge independent and stabilize the results with respect to the variation of the scale  $Q$  to which the relevant RGE is integrated from the high energy scale of the see-saw mechanism, thus clarifying the points raised in [11].

The general formulae for the corrections  $I_{AB}^{\text{th}}$  derived in this paper can be applied to various models predicting the neutrino masses and mixing. They can be used to quantify the slepton mass splitting and/or the amount of flavor violation in the slepton sector necessary to realize the specific mechanisms, investigated in papers [8–10], allowing to obtain correct mass squared differences and mixing angles from initially equal neutrino masses. They can find particularly interesting application in concrete models [18] relating the see-saw mechanism generating neutrino masses to flavor non-conservation in the slepton sector. Finally, they will be indispensable for future precision tests of any quantitative theory of the neutrino masses.

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